Topology

B. Math. II

Mid-Term Examination

Instructions: All questions carry equal (non-zero) marks.

1. Let $\mathbb{R}^\omega$ denote the countably infinite product of the set of real numbers. Define box and product topology on $\mathbb{R}^\omega$. Let $\mathbb{R}^\infty$ denote the subset of $\mathbb{R}^\omega$ consisting of all sequences which are zero except possibly for finitely many entries. Find closure of this subset in box as well as product topology. What conclusions can you draw about these two topologies by looking at these closures?

2. Define a Hausdorff space. Show that a topological space X is Hausdorff if and only if the diagonal $\Delta = \{(x, x) \mid x \in X\}$ is closed in $X \times X$.

3. Let X and Y be connected spaces and let $A \subset X$ and $B \subset Y$ be **proper** subsets. Prove that
$$(X \times Y) \setminus (A \times B)$$
is connected.

4. Define **component** and **path component** of a space X and also define the **lower limit topology** on the set of real numbers. We denote this topological space by $\mathbb{R}_l$. What are the components and path components of $\mathbb{R}_l$? Describe continuous maps $f : \mathbb{R}_{std} \to \mathbb{R}_l$.

5. Let X be a Hausdorff space and let A and B be two **disjoint** compact subspaces of X. Prove that there exist **disjoint** open set U and V of X such that $A \subset U$ and $B \subset V$.

6. Prove that finite product of compact spaces is compact.